Five-minute review

1. Why is t critical value often larger than z?

2. n=25, known σ=10, 95% E?

Write your answers before we discuss them.

Hypotheses, tails and errors

What we will learn

Formulate null and alternative hypotheses and distinguish testing errors.

Why does it matter?

A statistical test asks whether the data provide enough evidence against a benchmark. Every decision has a possibility of error.

Definitions: Hypotheses tails and errors

Null hypothesis H_0: benchmark statement tested against the data.

Alternative hypothesis H_a: the competing statement defining the direction of evidence.

Significance level α: the chosen probability of a Type I error under a calibrated simple null.

Formulas

$$\text{Power} = 1 - \beta$$

Important ideas and notation

Null H_0 gives a benchmark or no-effect condition. Alternative H_a gives the directional or two-sided research question. Equality belongs in H_0; the claim may be either hypothesis.

Choose significance α before looking at results. Reject H_0 when evidence is sufficiently extreme; otherwise fail to reject. Not rejecting does not establish truth.

Using the method correctly

Type I error: reject true H_0 (probability α for a calibrated simple null).
Type II: fail to reject false H_0 (β at a specified alternative). Power=$1-\beta$.

Example 7.1.1

Claim average fill exceeds 500 mL. State H_0 and H_a and identify the tail.

Solution:

H_0: $\mu=500$ (boundary of $\mu \leq 500$)

H_a: $\mu>500$. Right-tailed.

Example 7.1.2

Claim defect rate differs from .05. State H_0 and H_a and identify the tail.

Solution:

$H_0: p = .05$

$H_a: p \neq .05$

two-tailed.

Example 7.1.3 • Quality control and decisions

If a good item is mistakenly flagged as defective, which error is it when H_0 is no defect?

Truth	Reject H_0	Fail to reject H_0
H_0 true	Type I	Correct
H_0 false	Correct	Type II

Illustrative classroom dataset

Example 7.1.3 • Solution

If a good item is mistakenly flagged as defective, which error is it when H_0 is no defect?

Solution:

Type I error.

Individual practice

1. Claim mean less than 20.

2. Reject H_0 when it is actually true: error?

3. If $\beta=.2$ at a specific alternative, power?

Group activity

For a screening alarm, define H_0 as no defect; describe both errors in context.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. H_0:$\mu=20$ (boundary of $\mu\geq20$); H_a:$\mu<20$.

2. Type I.

3. .8.

Group solution

Solution:

Type I: false alarm

Type II: miss a true defect.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. Can failure to reject prove equality?

2. A company claims mean battery life is 12 hours. Test whether it is lower. Write both hypotheses and identify the tail.

Show your reasoning. Submit before leaving.

Five-minute review

1. Can failure to reject prove equality?

2. Claim mean less than 20.

Write your answers before we discuss them.

One-proportion z test: critical values and p-values

What we will learn

Use both test decision methods on the same proportion problem.

Why does it matter?

The null value determines the standard error in a one-proportion z test. The alternative determines which tail we use.

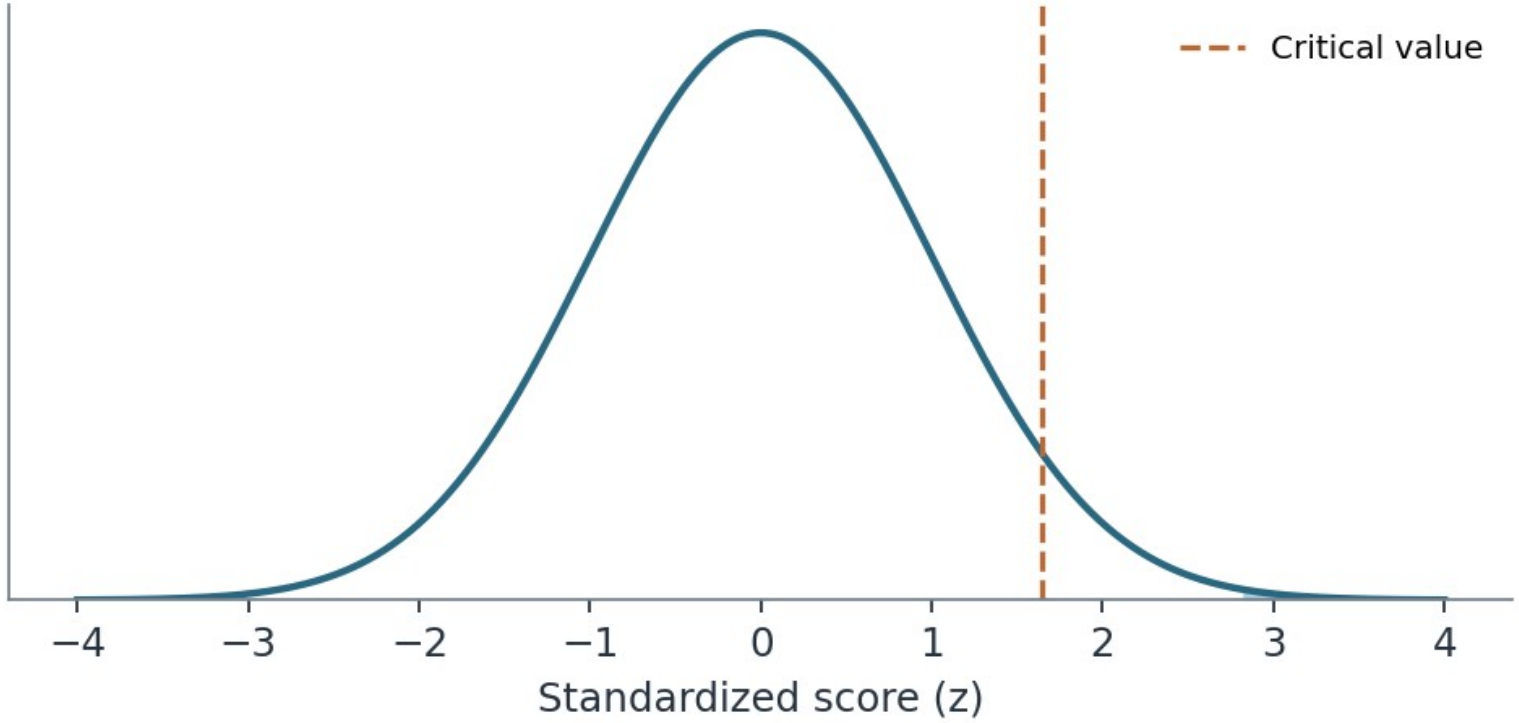
Definitions: One proportion z test critical values and p values

One-proportion z test: compares a sample proportion with a hypothesized population proportion using a standard normal reference.

Test statistic: standardized distance from the null benchmark.

P-value: probability, assuming the null, of a result at least as extreme in the direction of the alternative.

Illustration and interpretation



Notice the pattern

The null value determines the standard error in a one-proportion z test. The alternative determines which tail we use.

Illustrative data / model

Formulas

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Important ideas and notation

For testing $H_0: p = p_0$, $z = (\hat{p} - p_0)/\sqrt{[p_0(1-p_0)/n]}$. Check random independence and $np_0 \geq 10$, $n(1-p_0) \geq 10$; otherwise consider exact binomial test.

At $\alpha = .05$, right cutoff $+1.645$, left -1.645, two-sided ± 1.96. Use sign of H_a for rejection region.

p-value: probability under H_0 of a statistic as extreme or more in the direction of H_a. For symmetric z: right $1 - \Phi(z)$, left $\Phi(z)$, two $2\Phi(-\text{abs}(z))$. Reject if p-value $\leq \alpha$.

Example 7.2.1

Random sample 200; 120 successes; H_a:p>.5; α=.05. Test the claim using the critical value and p-value methods.

Solution:

H_0:p=.5

$\hat{p}$=.60

SE=.03536

z=2.828>1.645

p≈.00234

reject. Evidence p>.5.

Example 7.2.2

Same sample, H_a:$p \neq .5$. Test the two-sided alternative at $\alpha = .05$.

Solution:

abs(z)=2.828>1.96

p≈.00468

reject

evidence p differs from .5.

Example 7.2.3 • Is support above half?

Is the observed proportion above or below the null?

Quantity	Value
Sample size	200
Support	120
Null proportion	0.50
Sample proportion	0.60

Illustrative classroom dataset

Example 7.2.3 • Solution

Is the observed proportion above or below the null?

Solution:

Above by 0.10

the right-tail test asks whether that is unusually large.

Individual practice

1. $n=100$, $x=50$, $p_0=.5$; z?

2. $z=-2.1$, left-tailed, $\alpha=.05$; decision?

3. $p=.08$, $\alpha=.05$; decision?

Group activity

$n=100, x=40, H_a: p<.5, \alpha=.05$. Give all steps.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. 0.

2. Reject; $-2.1 < -1.645$.

3. Fail to reject H_0.

Group solution

Solution:

H_0:p=.5

SE=.05

z=−2

critical −1.645

p≈.0228

reject

evidence p<.5.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. Which proportion appears in test SE?

2. For a two-sided z test, $z=-2.10$ and $\alpha=.05$. Use critical values ± 1.96. Decide.

Show your reasoning. Submit before leaving.

Five-minute review

1. Which proportion appears in test SE?

2. n=100, x=50, p_0=.5; z?

Write your answers before we discuss them.

One-mean z test

What we will learn

Test a population mean with known population standard deviation.

Why does it matter?

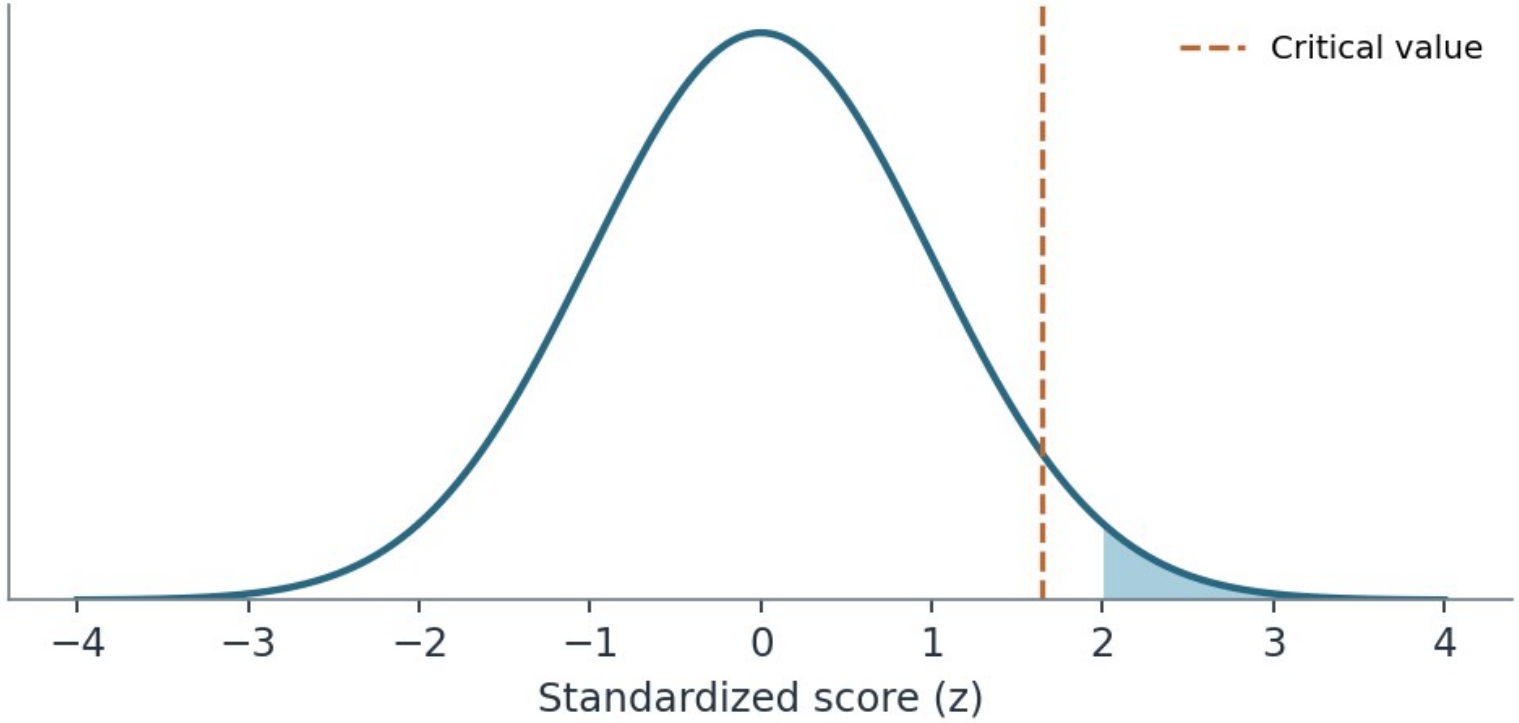
A known population SD lets us standardize a sample mean. The standard error, not the individual SD, is the denominator.

Definitions: One mean z test

One-mean z test: tests a population mean when the population SD is known and sampling/model conditions hold.

Critical region: values of the test statistic that lead to rejection of H_0.

Illustration and interpretation



Notice the pattern

A known population SD lets us standardize a sample mean. The standard error, not the individual SD, is the denominator.

Illustrative data / model

Formulas

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Important ideas and notation

Known σ and independent random sample: $z=(\bar{x}-\mu_0)/(\sigma/\sqrt{n})$. Require normal population or sufficient sample size for approximate normality.

Five steps: hypotheses and α; check conditions and calculate statistic; find critical value or p-value; decision; contextual conclusion.

Choose right/left/two-sided tail from H_a. Report effect estimate and distinguish practical importance from significance.

Example 7.3.1

$n=36$, $\bar{x}=102$, $\sigma=6$, $H_a:\mu>100$, $\alpha=.05$. Test the claim using a significance level of .05.

Solution:

$H_0:\mu=100$

SE=1

z=2

cutoff 1.645

p=.0228

reject, evidence mean>100.

Example 7.3.2

Same data, H_a:$\mu \neq 100$. Test the two-sided alternative at $\alpha = .05$.

Solution:

$z = 2$

$|z| > 1.96$

$p \approx .0455$

reject at .05.

Example 7.3.3 • Checking a package label

How many standard errors above the null mean is 102?

Quantity	Value
Claimed mean	100
Sample mean	102
Population SD	6
Sample size	36

Illustrative classroom dataset

Example 7.3.3 • Solution

How many standard errors above the null mean is 102?

Solution:

$(102-100)/(6/\sqrt{36}) = 2.$

Individual practice

1. $n=25$, $\sigma=10$: SE?

2. $\bar{x}=48$, $\mu_0=50$, SE=2: z?

3. $z=1$, right-tailed $\alpha=.05$: decision?

Group activity

$n=64, \bar{x}=49, \sigma=8$, $H_a: \mu<50$, $\alpha=.05$.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. 2.

2. −1.

3. Fail to reject.

Group solution

Solution:

SE=1

z=−1

p≈.1587

fail to reject

insufficient evidence mean<50.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. When is one-mean z test appropriate rather than t?

2. An independent sample of 36 has mean 51. Known $\sigma=6$; test H_0: $\mu=50$ versus $\mu>50$ at .05.

Show your reasoning. Submit before leaving.

Five-minute review

1. When is one-mean z test appropriate rather than t?

2. n=25, σ=10: SE?

Write your answers before we discuss them.

One-mean t test and interpreting p-values

What we will learn

Use a one-sample t test when population SD is unknown.

Why does it matter?

The t distribution accounts for estimating variability. Small samples require particular care with outliers and distribution shape.

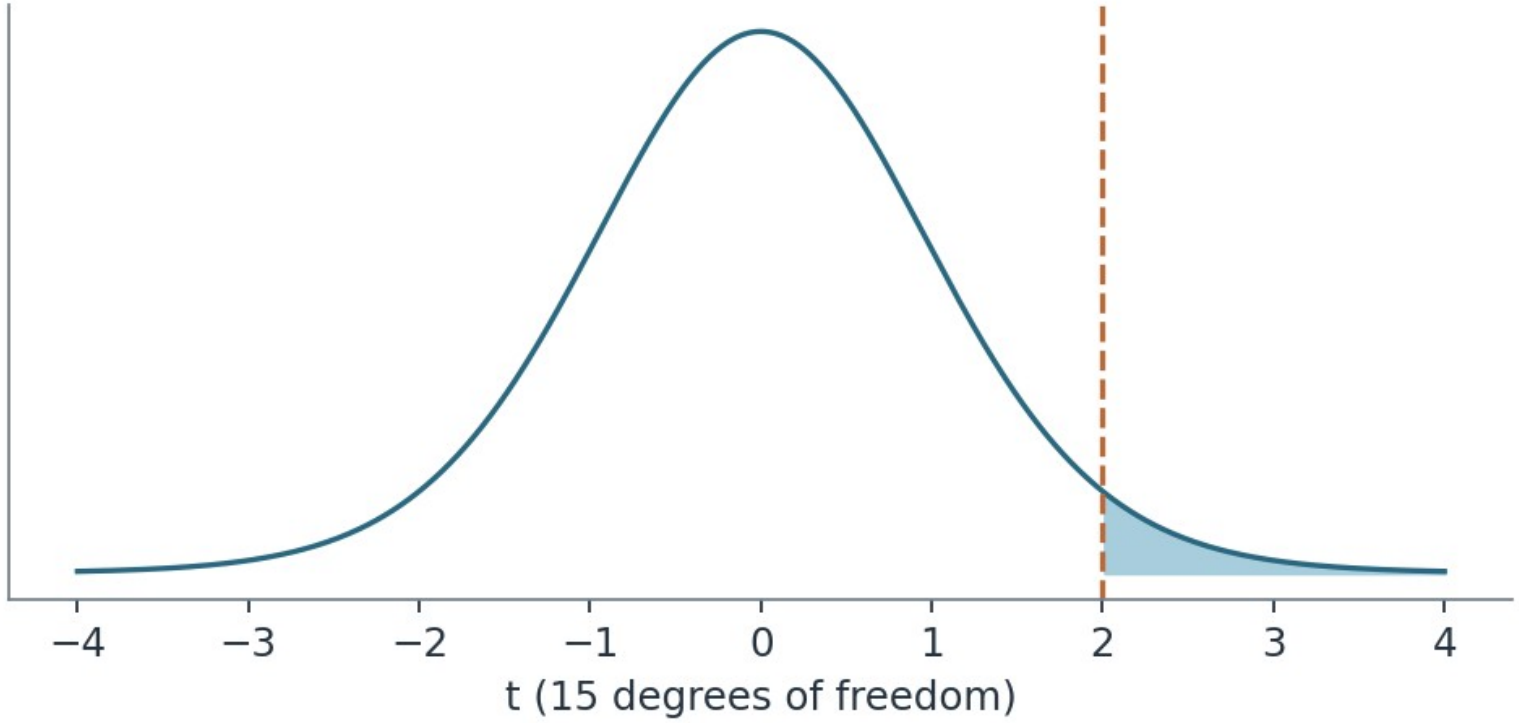
Definitions: One mean t test and interpreting p values

One-mean t test: compares a sample mean with a benchmark using sample SD and a t reference distribution.

Degrees of freedom: $n-1$ in the one-sample t test.

Statistical significance: a result meets the prespecified rejection criterion, not necessarily practical importance.

Illustration and interpretation



Notice the pattern

The t distribution accounts for estimating variability. Small samples require particular care with outliers and distribution shape.

Illustrative data / model

Formulas

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}, \quad df = n - 1$$

Important ideas and notation

For independent random observations and normal/approximately normal sampling mean, $t=(\bar{x}-\mu_0)/(s/\sqrt{n})$, $df=n-1$.

In R, use pt(t,df) for lower tail, pt(t,df,lower.tail=FALSE) for upper tail, 2*pt(-abs(t),df) for two-sided.

A p-value is not $P(H_0$ is true), not effect size and not probability of replication. Compare with a prespecified α.

Example 7.4.1

$n=16, \bar{x}=54, s=8$, $H_a: \mu > 50$, $\alpha=.05$. Test the claim at $\alpha=.05$.

Solution:

$t=2$, $df=15$

critical 1.753

$p \approx .0320$

reject

evidence mean>50.

Example 7.4.2

Same data, two-sided H_a:$\mu \neq 50$. Test the two-sided alternative at $\alpha = .05$.

Solution:

critical ± 2.131

$p \approx .0640$

fail to reject

insufficient evidence of difference.

Example 7.4.3 • A small sample and an unknown SD

Compute t and degrees of freedom.

Quantity	Value
Sample size	16
Sample mean	54
Sample SD	8
Null mean	50

Illustrative classroom dataset

Example 7.4.3 • Solution

Compute t and degrees of freedom.

Solution:

$(54-50)/(8/\sqrt{16})=2$

$df=15.$

Individual practice

1. n=10: df?

2. $\bar{x}=18$, $\mu_0=20$, $s=4$, $n=16$: t?

3. $p=.049$, $\alpha=.01$: decision?

Group activity

$n=9, \bar{x}=12, s=3$, $H_a: \mu > 10$, $\alpha = .05$; normal population.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. 9.

2. -2.

3. Fail to reject.

Group solution

Solution:

t=2,df=8

cutoff 1.860

p≈.0403

reject.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. Does a small p-value measure how large a difference is?

2. A t test produces p=.08 at α=.05. What is the correct decision?

Show your reasoning. Submit before leaving.