Five-minute review

1. Why should we check the shape before using a normal probability table?

2. A bell-shaped distribution has mean 50 and SD 5. Estimate the fraction between 40 and 60.

Write your answers before we discuss them.

Point estimates, confidence and margin of error

What we will learn

Explain point estimates and confidence intervals without confusing confidence with probability of a fixed parameter.

Why does it matter?

A point estimate alone does not convey how much sampling variation to expect. An interval adds that information.

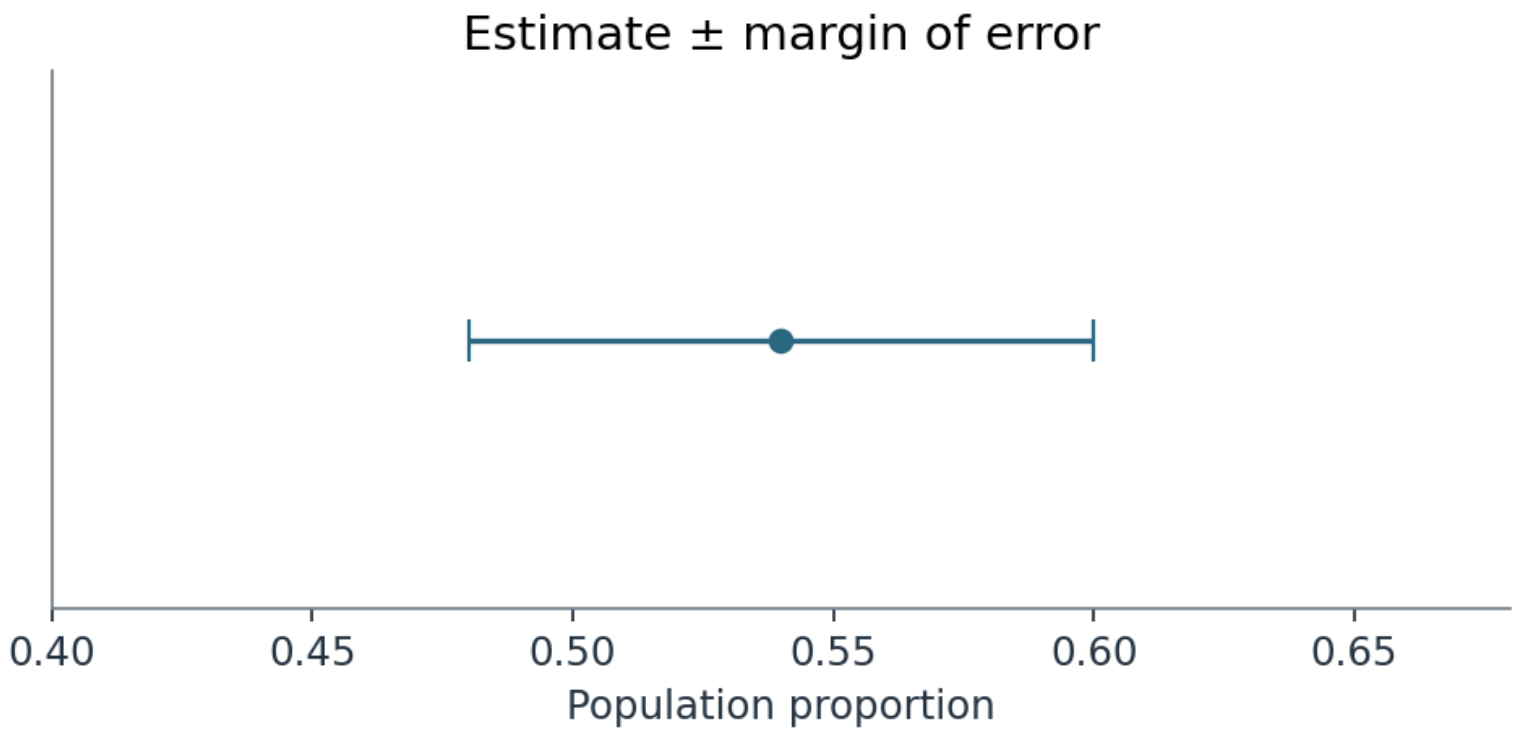
Definitions: Point estimates confidence and margin of error

Point estimate: a single sample-based estimate of a population parameter.

Confidence interval: a range generated by a statistical procedure to estimate the parameter.

Confidence level: long-run coverage of that procedure, under its assumptions.

Illustration and interpretation



Notice the pattern

A point estimate alone does not convey how much sampling variation to expect. An interval adds that information.

Illustrative data / model

Formulas

$$\hat{p} = \frac{x}{n}, \quad E = \frac{U - L}{2}$$

$$\text{Interval} = \text{Estimate} \pm E$$

Important ideas and notation

Point estimates: $\hat{p}=x/n$ for population proportion p, $\bar{x}$ for μ, s^2 for σ^2. An interval estimate adds uncertainty around a point estimate.

For symmetric intervals, estimate ± margin of error E; $E=(\text{upper}-\text{lower})/2$. Confidence level $1-\alpha$ is the long-run coverage rate of the procedure under its assumptions.

Common two-sided normal critical values: 90% 1.645; 95% 1.960; 99% 2.576. Higher confidence generally gives wider intervals at fixed n.

Example 6.1.1

A 95% interval is (0.48,0.60). Find the point estimate and margin of error. Interpret the confidence level.

Solution:

Point estimate .54

margin .06. The procedure captures the fixed parameter in about 95% of repeated samples, not a 95% probability for this particular fixed parameter.

Example 6.1.2

For confidence .90, find α and each tail.

Solution:

α=.10

each tail .05

z*=1.645.

Example 6.1.3 • A survey estimate with uncertainty

Does this interval guarantee the population proportion is inside?

Point estimate	Margin	Interval
54%	6 points	48% to 60%

Illustrative classroom dataset

Example 6.1.3 • Solution

Does this interval guarantee the population proportion is inside?

Solution:

No. A 95% confidence procedure captures the fixed parameter in about 95% of repeated samples under its conditions.

Individual practice

1. Confidence 99%: α?

2. Interval (12,18): estimate and margin?

3. Which is wider at same n and variability, 90% or 95%?

Group activity

Simulate 20 repetitions of a sampling procedure or draw hypothetical intervals around known μ=50; count intervals containing 50.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. .01.

2. 15 and 3.

3. 95%.

Group solution

Solution:

Count varies

approximately $\frac{19}{20}$ on average for a valid 95% method, not guaranteed.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. Does 95% confidence mean 95% of observations lie within the interval?

2. A symmetric interval is (32,44). Find the point estimate and margin of error.

Show your reasoning. Submit before leaving.

Five-minute review

1. Does 95% confidence mean 95% of observations lie within the interval?

2. Confidence 99%: α?

Write your answers before we discuss them.

Confidence interval for one proportion

What we will learn

Construct and interpret a large-sample one-proportion confidence interval.

Why does it matter?

In a random sample, the observed proportion estimates the population proportion. A margin of error describes sampling uncertainty under the model.

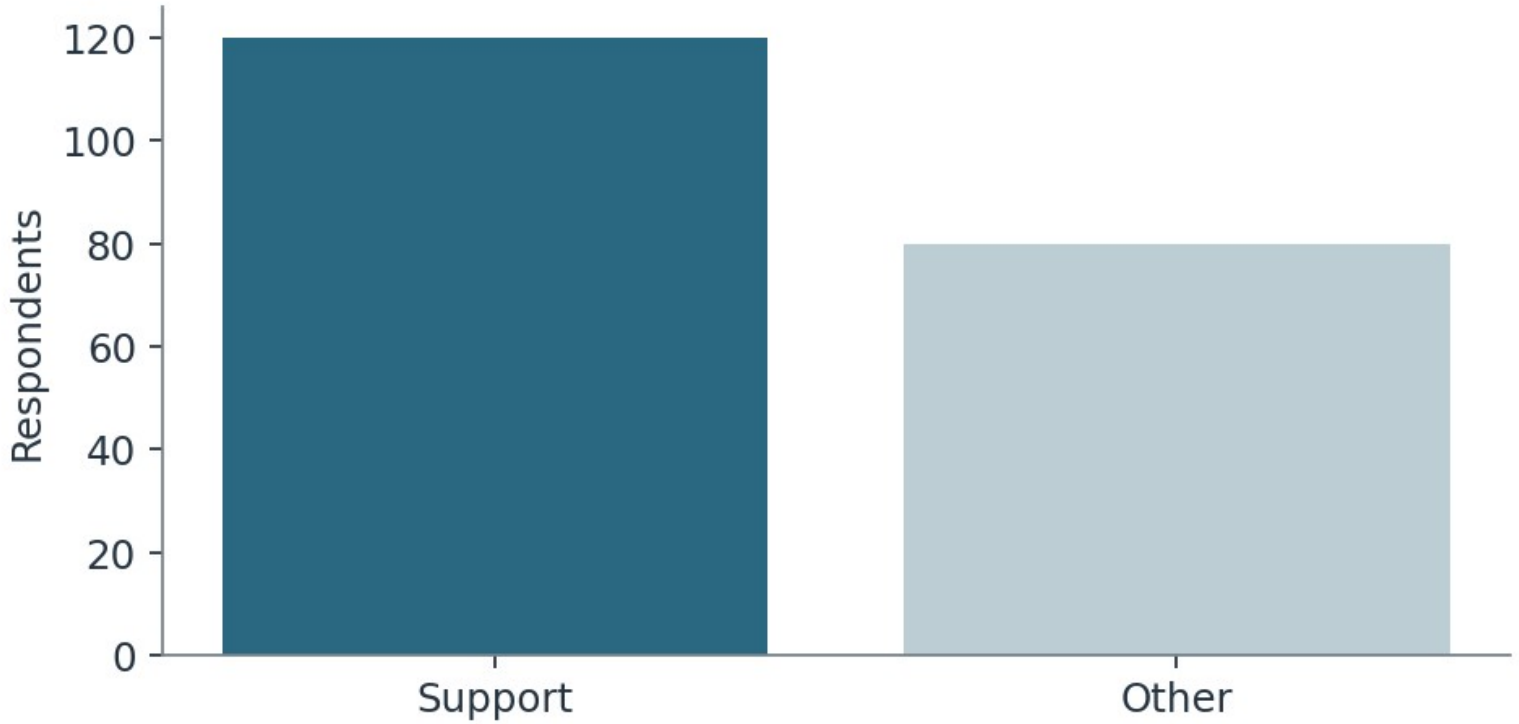
Definitions: Confidence interval for one proportion

Population proportion p: fraction of the population with a characteristic.

Sample proportion $\hat{p}$: number of observed successes divided by n.

Margin of error: critical value multiplied by the estimated standard error for this interval.

Illustration and interpretation



Notice the pattern

In a random sample, the observed proportion estimates the population proportion. A margin of error describes sampling uncertainty under the model.

Illustrative data / model

Formulas

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Important ideas and notation

For $\hat{p}=x/n$, $SE \approx \sqrt{[\hat{p}(1-\hat{p})/n]}$. Approximate Wald interval: $\hat{p} \pm z^*SE$.

Check random sampling, independence (10% guideline for sampling without replacement), and at least 10 successes and 10 failures. Near boundaries or small counts prefer Wilson or exact methods.

Interpret endpoints in context; a narrow interval is not evidence of absence of bias. Do not confuse α with margin of error.

Example 6.2.1

Random sample 200; 120 support a proposal. 95% CI. Calculate the approximate 95% confidence interval.

Solution:

$\hat{p}=.60$

$SE=\sqrt{(\frac{\cdot 24}{200})}\approx .03464$

$E=.06790$

$CI\approx(.5321,.6679)$.

Example 6.2.2

Sample 100; 50 successes, 90% CI. Calculate the approximate 90% confidence interval.

Solution:

$\hat{p}=.50$

SE=.05

E=.08225

CI≈(.4178,.5823).

Example 6.2.3 • Support for a proposal

What is the sample proportion?

Response	Count
Support	120
Do not support	80
Total	200

Illustrative classroom dataset

Example 6.2.3 • Solution

What is the sample proportion?

Solution:

$$\frac{120}{200} = 0.60.$$

Individual practice

1. Sample 400; 200 successes, 95% E?

2. Sample 100; 3 successes. Is Wald suitable?

3. What parameter does the interval estimate?

Group activity

Compare a 95% CI for 60/100 with 240/400.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. $1.96\sqrt{(\frac{\cdot 25}{400})}=.049.$

2. No; 3 successes <10.

3. Population proportion p.

Group solution

Solution:

Same $\hat{p}=.6$

$E \approx .0960$ and $.0480$

larger sample halves margin.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. Why can't an unbiased interval rescue a biased sample?

2. A random independent sample of 100 yields 60 successes. Find the approximate 95% Wald interval.

Show your reasoning. Submit before leaving.

Five-minute review

1. Why can't an unbiased interval rescue a biased sample?

2. Sample 400; 200 successes, 95% E?

Write your answers before we discuss them.

Planning sample size

What we will learn

Determine minimum sample size for a target margin of error.

Why does it matter?

Narrower margins require larger samples. Use a planning proportion of 0.5 when no reliable estimate is available.

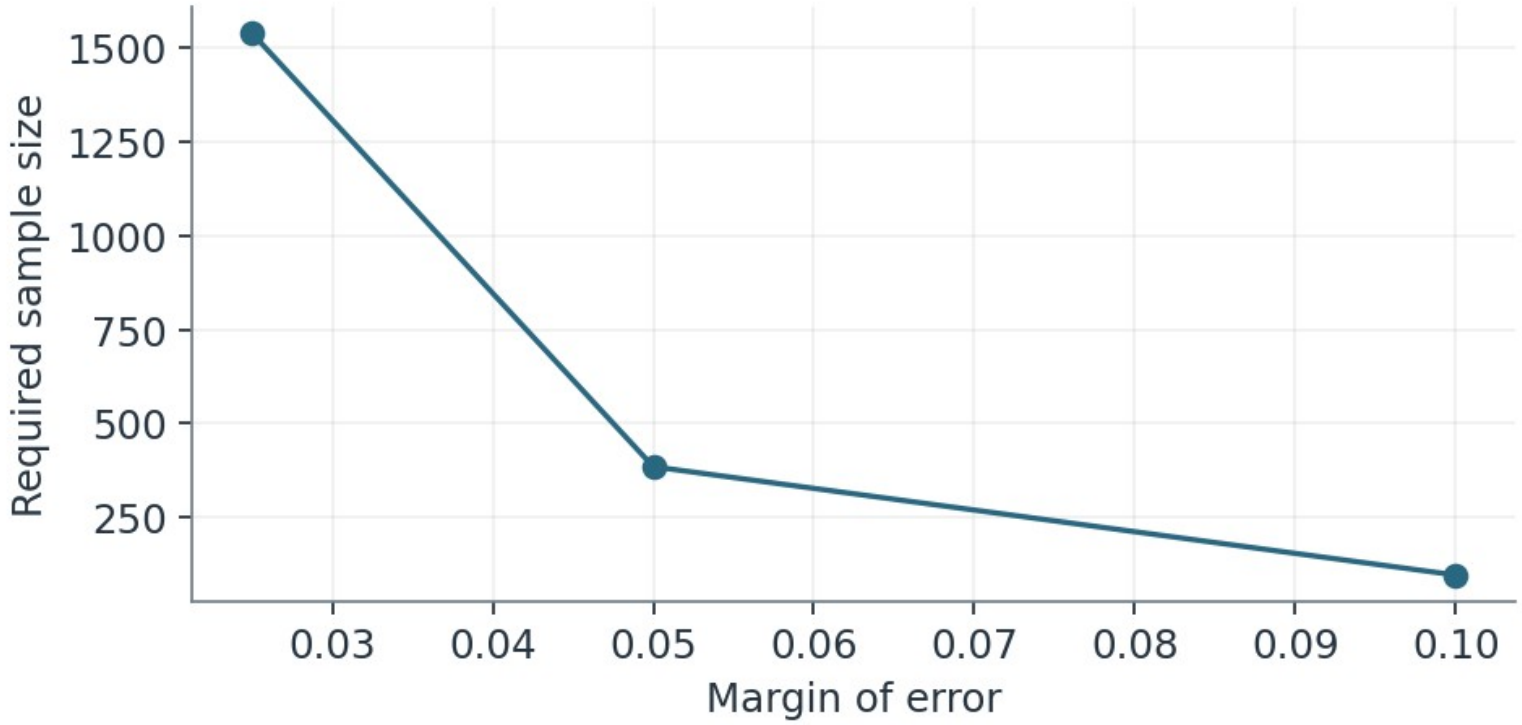
Definitions: Planning sample size

Sample size n: number of observations included in a study.

Target margin of error E: desired half-width of the confidence interval.

Planning value: an estimate used before data collection, such as p*=.5 when no prior proportion is known.

Illustration and interpretation



Notice the pattern

Narrower margins require larger samples. Use a planning proportion of 0.5 when no reliable estimate is available.

Illustrative data / model

Formulas

$$n \geq \frac{(z^*)^2 p^* (1 - p^*)}{E^2}$$

$$n \geq \left(\frac{z^* \sigma}{E}\right)^2$$

Important ideas and notation

For proportion with planning p^*, $n \geq (z^*/E)^2 \, p^*(1-p^*)$. If unknown, use .5 for conservative planning.

For estimating mean with a known or planning σ, $n \geq (z^*\sigma/E)^2$. These formulas assume independent observations and normal approximation.

Always round up to the next whole number; consider finite-population adjustment and nonresponse if relevant.

Example 6.3.1

95% confidence, proportion E=.05, no prior estimate. Find the minimum required sample size.

Solution:

$n \geq 1.96^2(.25)/.05^2 = 384.16$

require 385.

Example 6.3.2

95% confidence, planning $\sigma=10$, mean margin $E=2$. Find the minimum required sample size.

Solution:

$$n \geq \left(1.96 \times \frac{10}{2}\right)^2 = 96.04$$

require 97.

Example 6.3.3 • Planning a survey before collecting data

Why does halving the margin require about four times the sample?

Margin of error	Approx. required n (95%, p*=.5)
0.10	97
0.05	385
0.025	1537

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Example 6.3.3 • Solution

Why does halving the margin require about four times the sample?

Solution:

The sample-size formula is proportional to $1/E^2$.

Individual practice

1. 90% confidence, E=.10, p*=.5.

2. If desired margin halves, approximate sample size factor?

3. Why use p*=.5 if unknown?

Group activity

Plan a 95% survey with E=.04, compare p*=.2 and .5.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. $1.645^2 \times \frac{.25}{.01} \approx 67.65$; $n = 68$.

2. 4.

3. It maximizes $p^*(1-p^*)$.

Group solution

Solution:

n=385 and 601 respectively (round up).

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. If formula gives 120.01, minimum n?

2. For 95% confidence, E=.10 and no prior proportion, calculate the minimum planned sample size.

Show your reasoning. Submit before leaving.

Five-minute review

1. If formula gives 120.01, minimum n?

2. 90% confidence, E=.10, p*=.5.

Write your answers before we discuss them.

Confidence intervals for a mean: z and t

What we will learn

Choose z or t and calculate a confidence interval for a population mean.

Why does it matter?

If the population SD is not known, estimate it with the sample SD and use a t critical value under the appropriate assumptions.

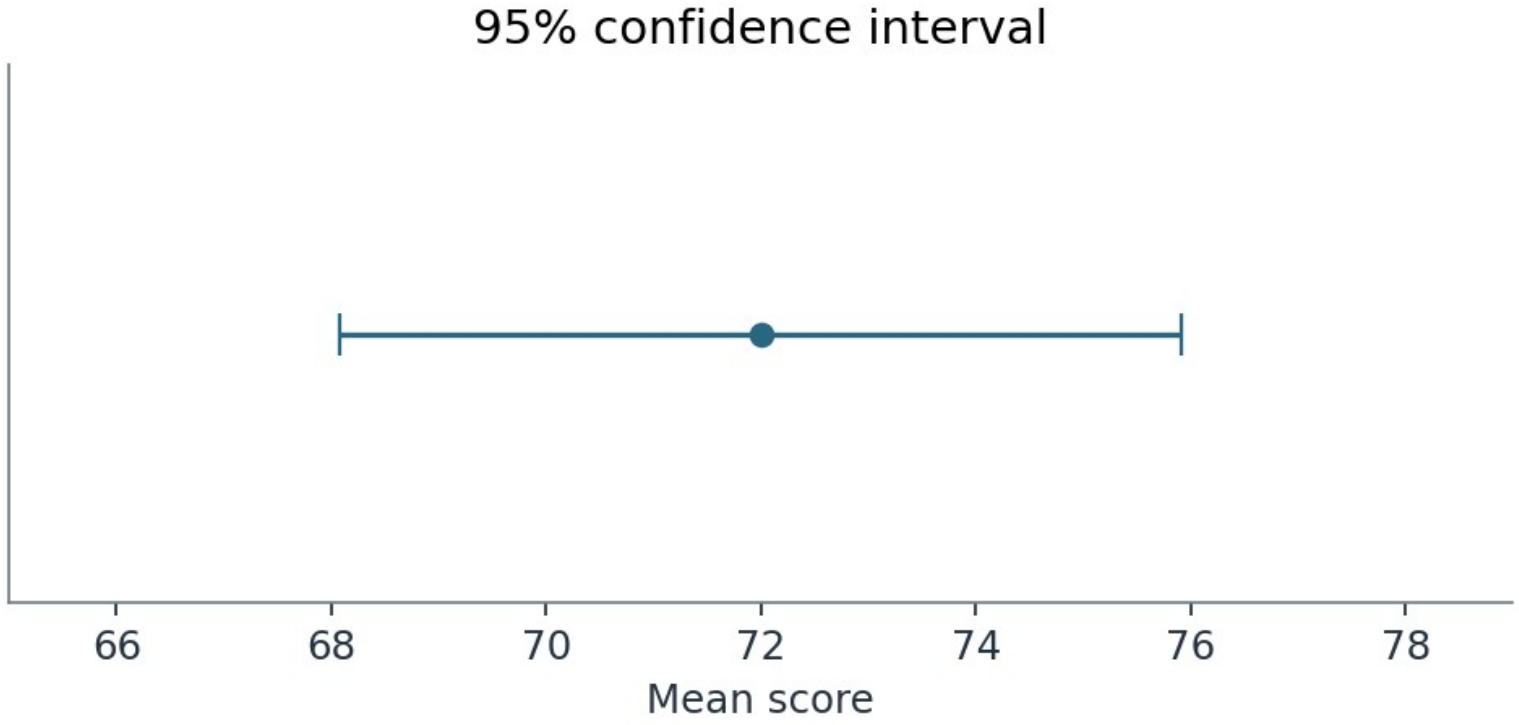
Definitions: Confidence intervals for a mean z and t

Confidence interval for a mean: an interval estimate for population mean μ.

Student t distribution: symmetric reference distribution with heavier tails than the standard normal at small degrees of freedom.

Degrees of freedom: $n-1$ for the usual one-sample t procedure.

Illustration and interpretation



Notice the pattern

If the population SD is not known, estimate it with the sample SD and use a t critical value under the appropriate assumptions.

Illustrative data / model

Formulas

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

Important ideas and notation

Known σ: $\bar{x} \pm z^* \sigma/\sqrt{n}$. Unknown σ: $\bar{x} \pm t^* s/\sqrt{n}$ with $df = n-1$. t accounts for estimating σ.

Require random independent observations and a normal population or sample large enough for an approximately normal mean; small samples need no strong skew or outliers.

t curves have heavier tails at small df; use R $qt(1-\alpha/2, df)$. Retain precision until final rounding.

Example 6.4.1

n=36, $\bar{x}$=72, known σ=12. 95% CI. Calculate the 95% z confidence interval.

Solution:

SE=2

E=3.92

CI=(68.08,75.92).

Example 6.4.2

n=16, $\bar{x}$=72, s=8, normal population. 95% t CI. Calculate the 95% t confidence interval.

Solution:

df=15

t*=2.131

SE=2

E≈4.262

CI≈(67.74,76.26).

Example 6.4.3 • Estimating a mean from sample data

What is the 95% z margin of error?

Quantity	Known SD example
Sample size	36
Sample mean	72
Population SD	12
SE	2

Illustrative classroom dataset

Example 6.4.3 • Solution

What is the 95% z margin of error?

Solution:

$1.96 \times 2 = 3.92$.

Individual practice

1. n=25, known σ=10, 95% E?

2. n=12, unknown σ. Which distribution and df?

3. If $\bar{x}$=40 and E=3, interval?

Group activity

Compare 95% z and t intervals at $n=9$, $\bar{x}=50$, σ or $s=6$.

Show your calculation or graph. Write a sentence explaining the result in context.

Check your answers

1. 3.92.

2. t, df=11.

3. 37 to 43.

Group solution

Solution:

z $E=3.92$

t $df=8$ $E\approx4.612$

t wider.

Compare your conclusion with the question. Check your units and explain what the numbers represent.

Exit ticket • 5 minutes

1. Why is t critical value often larger than z?

2. An independent sample from a normal population has n=25, mean 30 and s=5. With t*=2.064, find the 95% interval.

Show your reasoning. Submit before leaving.